Sādhanā Vol. 36, Part 3, June 2011, pp. 411–423. © Indian Academy of Sciences

A two-dimensional problem for a fibre-reinforced anisotropic thermoelastic half-space with energy dissipation

IBRAHIM A ABBAS

Department of mathematics, Faculty of Science, Sohag University, Sohag, 82524, Egypt
Department of mathematics, Faculty of Science, Jazan University, Jazan, 82822, Saudi Arabia
e-mail: ibrabbas7@yahoo.com

MS received 5 May 2010; revised 3 November 2010; accepted 4 March 2011

Abstract. The theory of thermoelasticity with energy dissipation is employed to study plane waves in a fibre-reinforced anisotropic thermoelastic half-space. We apply a thermal shock on the surface of the half-space which is taken to be traction free. The problem is solved numerically using a finite element method. Moreover, the numerical solutions of the non-dimensional governing partial differential equations of the problem are shown graphically. Comparisons are made with the results predicted by Green–Naghdi theory of the two types (GNII without energy dissipation) and (GNIII with energy dissipation). We found that the reinforcement has great effect on the distribution of field quantities. Results carried out in this paper can be used to design various fibre-reinforced anisotropic thermoelastic elements under thermal load to meet special engineering requirements.

Keywords. Fibre-reinforced; finite element method; thermoelasticity with energy dissipation.

1. Introduction

Fibre-reinforced composites are used in a variety of structures due to their low weight and high strength. The mechanical behaviour of many fibre-reinforced composite materials is adequately modelled by the theory of linear elasticity for transversely isotropic materials, with the preferred direction coinciding with the fibre direction. In such composites the fibres are usually arranged in parallel straight lines. The characteristic property of a reinforced composite is that its components act together as a single anisotropic unit as long as they remain in the elastic condition. Sengupta & Nath (2001) discussed the problem of surface waves in fibre-reinforced anisotropic elastic media. Singh (2001) showed that, for wave propagation in fibre-reinforced anisotropic media, this decoupling cannot be achieved by the introduction of the displacement potentials. Hashin & Rosen (1964) gave the elastic moduli for fibre-reinforced materials. In classical dynamical coupled theory of thermoelasticity, the thermal and mechanical waves propagate with an infinite velocity, which is not physically admissible.

The theory of couple thermoelasticity was extended by Lord & Shulman (1967) and Green & Lindsay (1972) by including the thermal relaxation time in constitutive relations. The theory was extended for anisotropic body by Dhaliwal & Sherief (1980). Green & Naghdi (1993) proposed a new generalized thermoelasticity theory by including the thermal-displacement gradient among the independent constitutive variables. An important feature of this theory, which is not present in other thermoelasticity theories, is that it does not accommodate dissipation of thermal energy. The relevant fundamental aspects of this theory are contained in Grenn & Naghdi (1991; 1992). Singh (2006) studied the wave propagation in thermally conducting linear fibre-reinforced composite materials with one relaxation time.

The exact solution of the governing equations of the generalized thermoelasticity theory for a coupled and nonlinear/linear system exists only for very special and simple initial and boundary problems. A numerical solution technique is used to calculate the solution of general problems. For this reason the finite element method is chosen.

The Finite element method is a powerful technique originally developed for numerical solution of complex problems in structural mechanics, and it remains the method of choice for complex systems. A further benefit of this method is that it allows physical effects to be visualized and quantified regardless of experimental limitations. On the other hand, the finite element method in different generalized thermoelastic problems has been applied by many authors (see for instance Tian *et al* 2006; Abbas 2007; Abbas & Abd-alla 2008; Abbas & Othman 2009; Youssef & Abbas 2007).

In the present work, the Green and Naghdi theory is applied to study the influence of reinforcement on the total deformation body and the interaction with each other. Furthermore, the problem is solved numerically using a finite element method (FEM). Numerical results for the temperature distribution, displacement and the stress components are represented graphically. As an application of the results carried out in this paper, we can design various fibre-reinforced anisotropic thermoelastic elements under thermal load to meet special engineering requirements.

2. Basic equations and formulation

According to Green & Nagdhi (1993) and Singh (2006), the linear equations governing thermoelastic interactions in homogeneous anisotropic solid in the absence of body force and heat sources are as follows:

$$\tau_{ij,j} = \rho \ddot{u}_i, \quad i, j = 1, 2, 3, \tag{1}$$

$$K^* T_{,ij} + K_{ij} \dot{T}_{,ij} = \rho c_e \ddot{T} + T_o \beta_{ij} \ddot{u}_{i,j}, \quad i, j = 1, 2, 3. \tag{2}$$

The constitutive equation for a fibre-reinforced linearly thermoelastic anisotropic medium whose preferred direction is that of a unit vector **a** will be as follows (Belfield *et al* (1983):

$$\begin{aligned} \tau_{ij} = {} & \lambda e_{kk}\delta_{ij} + 2\mu_T e_{ij} + \alpha\left(a_k a_m e_{km}\delta_{ij} + a_i a_j e_{kk}\right) \\ & + 2\left(\mu_L - \mu_T\right)\left(a_i a_k e_{kj} + a_j a_k e_{ki}\right) \\ & + \beta\, a_k a_m\, e_{km} a_i a_j - \beta_{ij}\left(T - T_0\right)\delta_{ij}, \quad i, j, k, m = 1, 2, 3, \end{aligned} \tag{3}$$

where ρ is the mass density; u_i the displacement vectors components; e_{ij} the strain tensor; τ_{ij} the stress tensor; T the temperature change of a material particle; T_o the reference uniform temperature of the body; β_{ij} the thermal elastic coupling tensor; c_e the specific heat at constant strain; K_{ij} the thermal conductivity; K^* the material characteristic of the theory; λ, μ_T are elastic parameters; α, β, $(\mu_L - \mu_T)$ are reinforced anisotropic elastic parameters and $\mathbf{a} \equiv (a_1, a_2, a_3)$,

$a_1^2 + a_2^2 + a_3^2 = 1$. The comma notation is used for spatial derivatives and superimposed dot represents time differentiation.

We consider the problem of a fibre-reinforced anisotropic half-space ($x_1 \geq 0$) All the considered functions will depend on the time t and the coordinates x_1 and x_2 Thus, the displacement vector u_i will have the components

$$u = u_1 = u(x_1, x_2, t), \; v = u_2 = v(x_1, x_2, t), \; w = u_3 = 0. \tag{4}$$

We choose the fibre-direction as $\mathbf{a} \equiv (1, 0, 0)$ so that the preferred direction is the x_1 axis and Eqs. (1), (2) and (3) are simplified to be

$$\tau_{11} = (\lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta)\frac{\partial u}{\partial x_1} + (\lambda + \alpha)\frac{\partial v}{\partial x_2} - \beta_{11}(T - T_o), \tag{5}$$

$$\tau_{22} = (\lambda + 2\mu_T)\frac{\partial v}{\partial x_2} + (\lambda + \alpha)\frac{\partial u}{\partial x_1} - \beta_{22}(T - T_o), \tag{6}$$

$$\tau_{12} = \mu_L\left(\frac{\partial v}{\partial x_1} + \frac{\partial u}{\partial x_2}\right), \tag{7}$$

$$A_{11}\frac{\partial^2 u}{\partial x_1^2} + A_{12}\frac{\partial^2 v}{\partial x_1 \partial x_2} + A_{13}\frac{\partial^2 u}{\partial x_2^2} - \beta_{11}\frac{\partial T}{\partial x_1} = \rho\frac{\partial^2 u}{\partial t^2}, \tag{8}$$

$$A_{22}\frac{\partial^2 v}{\partial x_2^2} + A_{12}\frac{\partial^2 u}{\partial x_1 \partial x_2} + A_{13}\frac{\partial^2 v}{\partial x_1^2} - \beta_{22}\frac{\partial T}{\partial x_2} = \rho\frac{\partial^2 v}{\partial t^2}, \tag{9}$$

$$\left(K^* + K_{11}\frac{\partial}{\partial t}\right)\frac{\partial^2 T}{\partial x_1^2} + \left(K^* + K_{22}\frac{\partial}{\partial t}\right)\frac{\partial^2 T}{\partial x_2^2} = \rho c_e\frac{\partial^2 T}{\partial t^2} + T_o\frac{\partial^2}{\partial t^2}\left(\beta_{11}\frac{\partial u}{\partial x_1} + \beta_{22}\frac{\partial v}{\partial x_2}\right), \tag{10}$$

with

$$A_{11} = \lambda + 2(\alpha + \mu_T) + 4(\mu_L - \mu_T) + \beta, \; A_{12} = \alpha + \lambda, \; A_{13} = \mu_L, \; A_{22} = \lambda + 2\mu_T,$$

$$\beta_{11} = (2\lambda + 3\alpha + 4\mu_L - 2\mu_T + \beta)\alpha_{11} + (\lambda + \alpha)\alpha_{22}, \; \beta_{22} = (2\lambda + \alpha)\alpha_{11} + (\lambda + 2\mu_T)\alpha_{22},$$

where α_{11}, α_{22} are coefficients of linear thermal expansion. For convenience, the following non-dimensional variables are used:

$$\left(x', y', u', v'\right) = c\chi(x, y, u, v), \; t' = c^2\chi t, \quad T' = \frac{\beta_{11}(T - T_0)}{\rho c^2},$$
$$\left(\tau'_{11}, \tau'_{22}, \tau'_{12}\right) = \frac{1}{\rho c^2}(\tau_{11}, \tau_{22}, \tau_{12}), \; c^2 = \frac{A_{11}}{\rho}, \; \chi = \frac{\rho c_e}{K_{11}}. \tag{11}$$

In terms of the non-dimensional quantities defined in Eqs. (11), the above governing equations reduce to (dropping the dashed for convenience)

$$\tau_{11} = \frac{\partial u}{\partial x_1} + B_1\frac{\partial v}{\partial x_2} - T, \tag{12}$$

$$\tau_{22} = B_1\frac{\partial u}{\partial x_1} + B_2\frac{\partial v}{\partial x_2} - B_3 T, \tag{13}$$